Published by Institute of Physics Publishing for SISSA

Received: *March 14, 2006*
Revised: *March 31, 2006*
Accepted: *April 10, 2006*
Published: *April 27, 2006*

Exploring the SO(32) heterotic string

Hans Peter Nilles,[a] Saúl Ramos-Sánchez,[a] Patrick Vaudrevange[a] and Akın Wingerter[b]

[a] *Physikalisches Institut, Universität Bonn*
Nussallee 12, D-53115 Bonn, Germany
[b] *Department of Physics, The Ohio State University*
191 W. Woodruff Ave., Columbus, OH 43210, U.S.A.
E-mail: nilles@th.physik.uni-bonn.de, ramos@th.physik.uni-bonn.de, patrick@th.physik.uni-bonn.de, akin@mps.ohio-state.edu

Abstract: We give a complete classification of $\mathbb{Z}_N$ orbifold compactification of the heterotic SO(32) string theory and show its potential for realistic model building. The appearance of spinor representations of SO($2n$) groups is analyzed in detail. We conclude that the heterotic SO(32) string constitutes an interesting part of the string landscape both in view of model constructions and the question of heterotic-type I duality.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua.

JHEP04(2006)050

Contents

1. Introduction

String theory might provide a framework to describe all particle physics phenomena. Still we do not know how to derive the standard model of strong and electro-weak interactions from first principles. Apparently many roads seem to be possible: the so-called landscape of string vacua. Progress might be made by exploring this landscape in detail to understand possible phenomenological patterns that might be mapped to experimental observations. Such patterns might include concepts like supersymmetry, grand unification and extra dimensions.

In the present paper we would like to explore the heterotic SO(32) string theory and its suitability for model building. There has been less effort spent on the SO(32) theory than its $\mathrm{E}_8 \times \mathrm{E}_8'$ brother, which was considered as the prime candidate initially. A detailed analysis of the SO(32) theory shows, however, that model building within this framework could be as exciting as in the $\mathrm{E}_8 \times \mathrm{E}_8'$ case.

An additional motivation to consider the SO(32) heterotic theory is the exploration of the conjectured duality to the SO(32) type I string theory [1]. This might prove useful to understand connections between heterotic model constructions and those based on type II orientifolds.

JHEP04(2006)050

Our analysis considers the orbifold compactification[2] of the SO(32) heterotic theory,[1] as it combines the complexity of Calabi-Yau compactification with the calculability of torus compactification. Many phenomenological properties find a geometric explanation in this framework [5 – 7]. We derive a complete classification of the four-dimensional heterotic SO(32) orbifold constructions. This is necessary as previous attempts to do so have been found to be incomplete. We explain the subtleties of the construction and give a detailed presentation of the $\mathbb{Z}_4$-orbifold. The remaining cases are given in detail on a web page [8] that will be made available to the public.

Having achieved this goal of classification we explore properties that might be important for explicit model building. One aspect e.g. is the question of the appearance of spinor representations of SO($2n$) gauge groups (with $n = 5, 6, 7$). Spinors of SO(10) [9, 10] e.g. would be very suitable for a description of families of quarks and leptons, as argued in ref. [11]. In addition, the appearance of these spinors might be relevant to understand the nature of the heterotic-type I duality in four space time dimensions [12 – 15].

Using this information we provide a few explicit examples of 3-family models in this framework to illustrate the ease with which such models can be constructed. One example is obtained even in the absence of Wilson lines. Our results can be used as a starting point for a full classification of models including Wilson lines, the inclusion of which is, however, beyond the scope of this paper. Nonetheless, some useful patterns of possible spectra can be deduced from our results with the concept of fixed-point equivalent models [16]. It thus appears that the heterotic SO(32) theory is a fertile part of the string theory landscape.

The paper is organized as follows. In section 2.1 we present the strategy to classify all orbifolds of the SO(32) heterotic string. In section 2.2 and 2.3 we illustrate the method for the $\mathbb{Z}_4$ orbifold explicitly and give the list of models for the $\mathbb{Z}_N$ orbifolds. Section 3 is devoted to the discussion of the spinorial representations of SO($2n$) gauge groups for various n. Two explicit examples of 3-family models will be presented in section 4, followed by concluding remarks in section 5. Some technical details and tables are given in the appendices.

2. Classification of orbifolds

2.1 Classification of SO(32) orbifold models

To introduce the relevant notation [17, 5] and to set the stage for the following calculations, we briefly summarize some of the concepts in orbifold constructions, before proceeding to describe the classification of inequivalent models.

An orbifold is defined to be the quotient of a torus[2] by a discrete set of its isometries, called the *point group* P. Modular invariance requires the action of the point group to be accompanied by a corresponding action G (*gauge twisting group*) on the 16 gauge degrees of freedom:

$$\mathcal{O} = T^6/P \otimes T^{16}/G\,. \tag{2.1}$$

[1] For earlier work on SO(32) heterotic string orbifolds, see ref. [3, 4].

[2] In a more general context, an orbifold is defined to be the quotient of a manifold by a discrete symmetry.

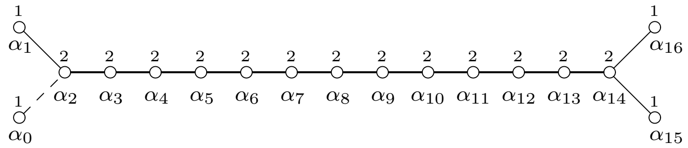


Figure 1: Extended Dynkin diagram of SO(32) and the associated Kač labels.

Modular invariance and the homomorphism property of the gauge embedding $P \hookrightarrow G$ put further restrictions on G, which will be discussed later. Consistency with ten-dimensional anomaly cancellation requires T^{16} to be an even, integral and self-dual lattice. In 16 dimensions, there are only 2 admissible choices, namely the root lattice of $\mathrm{E}_8 \times \mathrm{E}_8'$ and the weight lattice of $\mathrm{Spin}(32)/\mathbb{Z}_2$. Here, we focus our attention on the latter case.

The representations of Spin(32) fall into 4 conjugacy classes, corresponding to the adjoint, vector, spinor and conjugate spinor representation, respectively [18, 19]. Two representations are said to be conjugate, if their weight vectors differ by an element of the root lattice Λ_R. Consequently, the weight lattice Λ_W can be written as the sum of 4 disjoint sublattices, given by the highest weight of the respective representation modulo Λ_R. By $\mathrm{Spin}(32)/\mathbb{Z}_2$ we shall understand the symmetry corresponding to the adjoint and spinor conjugacy classes, and denote the respective lattice by $\Lambda_{\mathrm{Spin}(32)/\mathbb{Z}_2}$.

The action of G on T^{16} can be described as a shift $X_L \mapsto X_L + V$ [20], which induces the transformations

$$\sigma_V(H_i) = H_i, \qquad \sigma_V(E_\alpha) = \exp\left(2\pi i\, \alpha \cdot V\right) E_\alpha \tag{2.2}$$

on the Cartan generators and step operators of SO(32), and these transformations clearly describe an automorphism of the algebra.[3] The automorphisms of semi-simple Lie algebras have been classified [21], and it is straightforward to obtain the corresponding shifts, as we will now describe.

Automorphisms of SO(32)

To this end, consider the extended Dynkin diagram of SO(32) given in figure 1. The numbers which have been adjoined to the nodes are the Kač labels k_i, which are by definition the expansion coefficients of the highest root α_H in terms of the simple roots, i.e.

$$\alpha_H = k_1\alpha_1 + \ldots + k_\ell \alpha_\ell, \tag{2.3}$$

where ℓ is the rank[4] of the algebra. For convenience, the Kač label of the *most negative root* $\alpha_0 \equiv -\alpha_H$ is set to $k_0 = 1$. Then, by a theorem due to Kač [21], all order-N automorphisms of an algebra up to conjugation are given by

$$\sigma_{s,m}(E_{\alpha_j}) = \mu\, \exp\left(2\pi i s_j/N\right) E_{\alpha_j}, \quad j = 0, \ldots, \ell, \tag{2.4}$$

[3]Note that the group SO(32) and its covering group Spin(32) share the same algebra.

[4]Clearly, $\ell = 16$ in our case, but for the time being, we want to keep the discussion general.

JHEP04(2006)050

where the sequence $s = (s_0, \dots, s_\ell)$ may be chosen arbitrarily subject to the conditions that the s_i are non-negative, relatively prime integers and

$$N = m \sum_{i=0}^{\ell} k_i s_i \,. \tag{2.5}$$

Hereby, μ is an automorphism of the Dynkin diagram, and m is the smallest integer such that $(\sigma_{s,m})^m$ is inner. Since in this context we are only interested in inner automorphisms, we set $\mu = \mathbb{1}$ and $m = 1$. Furthermore, it should be noted that two automorphisms σ_s and $\sigma_{s'}$ are conjugate if and only if the sequence s can be transformed into the sequence s' by a symmetry of the extended diagram. In section 2.2, we will encounter an interesting example which shows that two such automorphisms of SO(32) must not be identified.

The shift vector

To derive the shift vector corresponding to a given automorphism is now particularly easy. Comparing eq. (2.2) to eq. (2.4), it immediately follows that

$$\alpha_i \cdot V = \frac{s_i}{N}, \quad i = 1, \dots, \ell, \tag{2.6}$$

for the ℓ linearly independent roots α_i. Expanding V in terms of the dual simple roots and substituting this expression in the previous equation gives

$$V = \frac{1}{N} \left(s_1 \alpha_1^* + \ldots + s_\ell \alpha_\ell^* \right), \tag{2.7}$$

i.e. the integers s_i divided by the order N are the Dynkin labels of V. It is checked by a direct calculation that this V also gives the correct transformation for the step operator corresponding to the most negative root.

Determining the unbroken gauge group is now particularly simple. Looking at eq. (2.7) we see that in the extended Dynkin diagram, the root α_i $(i = 0, \dots, \ell)$ is projected out, if and only if the coefficient s_i in eq. (2.5) does not vanish. To calculate the spectrum of the orbifold, we need an explicit expression for the shift vector V, which is easily obtained once the simple roots and their duals are given. For a standard choice of roots, see e.g. ref. [18].

Restrictions on the shift vector

Not every shift vector V which describes an automorphism of the algebra is an admissible choice for model construction. For a twist $\theta \in P$ of order N, $\theta^N = \mathbb{1}$ implies that $N\,V$ should act as the identity on T^{16}, and hence, from the self-duality of the lattice, it immediately follows that

$$N\,V \in \Lambda_{\mathrm{Spin}(32)/\mathbb{Z}_2} \,. \tag{2.8}$$

By eq. (2.7), $N\,V$ is only guaranteed to lie in the weight lattice Λ_W, so that some of the shift vectors will be ruled out.

For the partition function of a $\mathbb{Z}_N$ orbifold to be modular invariant, the relation

$$N \left(V^2 - v^2 \right) = 0 \bmod 2 \tag{2.9}$$

has to be satisfied [20], where v is a 3-dimensional vector describing the action of the twist on the complexified, compact coordinates. This condition severely restricts the number of shift vectors which can be used in constructing orbifold models.

From eq. (2.8) it is clear that for a given order N of the twist θ, all shifts V of order M are also admissible, as long as M divides N. In principle, we could determine the admissible shifts for each M separately, but a more practical approach is to run through the outlined procedure for N, dropping the condition on the relative-primeness of the sequence $s = (s_0, \ldots, s_\ell)$, see the remarks preceding eq. (2.5). In the cases where s is not relatively prime and the common divisor can be cancelled out from both the numerator and the denominator in eq. (2.7), the order of the shift is some M which is smaller than N.

We will illustrate the outlined methods using the $\mathbb{Z}_4$ orbifold in section 2.2.

2.2 The $\mathbb{Z}_4$ orbifold

Classification

We shall use the method presented in section 2.1 to compute all admissible shifts for the $\mathbb{Z}_4$ orbifold. For $N = 4$, there are 256 different vectors $s = (s_0, \ldots, s_{16})$, which satisfy eq. (2.5) with m=1 and the Kač labels k_i given in figure 1. We express the corresponding shift vectors using eq. (2.7) and a standard choice of roots[18]. Of these shift vectors, 134 satisfy the first restriction eq. (2.8) for admissible orbifold shifts and only 30 are left when we impose the modular invariance requirement, given by eq. (2.9) with the twist $v = \frac{1}{4}(1, 1, \text{-}2)$. Considering two shifts to be inequivalent if their spectra are different, we find only 16 inequivalent shift vectors in the $\mathbb{Z}_4$ orbifold. These are all possible shifts one can obtain.

Anomalies

The 16 inequivalent shift vectors, their corresponding gauge groups and spectra are listed in table 4 of appendix A. We have denoted the anomalous U_1 factors by U_{1A}. As a cross-check for our calculations, we have verified the following conditions for anomaly cancellation

$$\frac{1}{24}\mathrm{Tr}\, Q_i = \frac{1}{6|t_i|^2}\mathrm{Tr}\, Q_i^3 = \frac{1}{2}\mathrm{Tr}\, l Q_i = \begin{cases} \frac{1}{2|t_j|^2}\mathrm{Tr}\, Q_j^2 Q_A \neq 0 & \text{if } i = A,\, j \neq A \\ 0 & \text{otherwise} \end{cases} \tag{2.10}$$

where l denotes the index of a given representation. Furthermore, t_i is the generator of the i-th U_1 factor that defines the charge Q_i as:

$$Q_i|p_{sh}\rangle_L = (t_i \cdot p_{sh})|p_{sh}\rangle_L \ , \tag{2.11}$$

where p_{sh} is the shifted $\mathrm{Spin}(32)/\mathbb{Z}_2$ lattice vector. In the case when eq. (2.10) does not vanish, these conditions guarantee that the anomalous U_1 is cancelled by the generalized Green-Schwarz mechanism [22 – 26], [15].

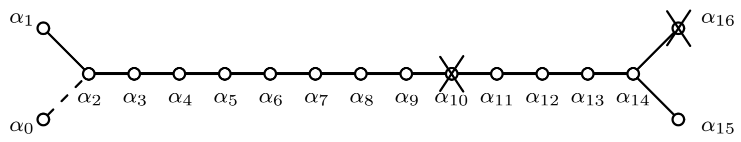
Discussion of the results

A detailed list including the spectra of all $\mathbb{Z}_4$ orbifold models is given in table 4 of appendix A. In particular, in the second column of this table we compare our results to those previously obtained in ref. [4]. In ref. [4] the shift vectors are separated into two classes. The so-called *vectorial* shifts in the $\mathbb{Z}_4$ orbifold are those whose entries have a maximal denominator of 4, whereas all entries of the *spinorial* shifts have a denominator of 8 and an odd numerator. Using these definitions, 12 of our shifts are vectorial and 4 are spinorial.

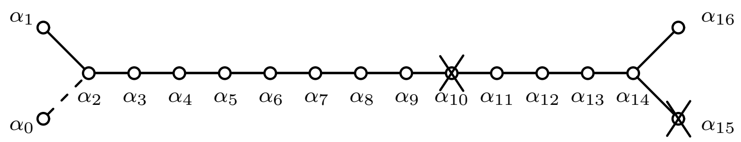
Our result differs in some ways from that of the ref. [4]. Some multiplicities of states and U(1) charges are different from our findings and cannot be related by a change of basis in the U(1) directions. For the models in question, ref. [4] does not fulfill the anomaly cancellation conditions, eq. (2.10).

Additionally, we find 16 inequivalent models whereas one can obtain only 10 inequivalent shifts with the general method proposed in ref. [4]. This discrepancy is related to two problems. One is that by using the ansatz for a spinorial shift proposed in ref. [4] and the weight lattice as given in section 2.1, one cannot obtain any of the four spinorial shifts we found in a direct manner for $\mathbb{Z}_4$. In appendix B we give an alternative ansatz for the form of any shift of $\mathbb{Z}_N$ orbifolds in the SO(32) heterotic string. Yet a classification based on this ansatz is more time consuming than the method presented in section 2.1.

The second problem is that the shift vectors $\mathrm{V}^{(4)}$ and $\mathrm{V}^{(12)}$ of our vectorial shifts are not listed in ref. [4]. Here, $\mathrm{V}^{(i)}$ denotes the shift vector corresponding to the model number i of table 4. The reason can be traced back to comparing, for instance, the shifts $\mathrm{V}^{(3)}$ and $\mathrm{V}^{(4)}$ of our list. Since both shifts generate the same unbroken gauge group in four dimensions and the same matter representations in the untwisted and second twisted sectors, one might be tempted to consider them to be equivalent. But the matter content of the first twisted sector is different, therefore, the two shifts lead to different models.



(a) Breaking due to $\mathrm{V}^{(3)}$



(b) Breaking due to $\mathrm{V}^{(4)}$

Figure 2: Extended Dynkin diagram of SO(32) corresponding to the breaking due to the shifts $\mathrm{V}^{(3)}$ and $\mathrm{V}^{(4)}$ of the $\mathbb{Z}_4$ orbifold.

JHEP04(2006)050

We have deeper reasons to argue that these two shifts are inequivalent. First, as illustrated in figure 2, the shift vectors come from two different breakings of the original SO(32) gauge group in ten dimensions. Two different breakings may be equivalent if one can transform the corresponding shifts into each other by adding lattice vectors and applying automorphisms of the lattice. We can see that there is no lattice vector relating the shifts $\mathrm{V}^{(3)}$ and $\mathrm{V}^{(4)}$. There exists an automorphism of SO(32), which maps $\mathrm{V}^{(3)}$ onto $\mathrm{V}^{(4)}$ up to a lattice vector, compare figure 2. It is not an automorphism of Spin(32)/$\mathbb{Z}_2$, as it transforms the *spinor* conjugacy class of the lattice of Spin(32)/$\mathbb{Z}_2$ into the *conjugate-spinor* class of Spin(32), which is not part of Spin(32)/$\mathbb{Z}_2$.

In summary, this shows that the ansatz of ref. [4] is incomplete. It leads only to 10 of the 16 shift vectors in the $\mathbb{Z}_4$ orbifold. As we shall show in section 4.1, one of the missing shifts leads to a three-family model.

2.3 The $\mathbb{Z}_N$ orbifold

Using the method described in section 2.1, we have computed all inequivalent models, which we do not list due to space limitations. All $\mathbb{Z}_N$ shifts, their corresponding gauge groups and spectra are listed on our web page [8].

Lists of models

In summary, there are 5141 $\mathbb{Z}_N$ orbifold models without Wilson lines. We have used the geometry of $\mathbb{Z}_N$ orbifolds as given in ref. [27], and for the $\mathbb{Z}_8$-I as given in ref. [28]. On our web site [8], we provide for each model the following details:

- the twist v and the 6 dimensional root lattice, which specifies the geometry,
- the gauge shift V and the corresponding gauge group,
- the matter content, listed by sectors, including all U_1 charges, where we have denoted the anomalous one by U_{1A}.[5]

For convenience, we have implemented a search engine, with which one can choose models with a given gauge group. As a side remark, this work can be seen as a contribution to the String Vacuum Project [29] in the context of the heterotic string [30].

Discussion of the results

In table 1 and table 2, we summarize our results. Our classification extends to SO(32) the results of ref. [31] obtained in the context of $\mathrm{E}_8 \times \mathrm{E}_8'$ heterotic orbifolds. Comparing the numbers of inequivalent SO(32) models to those presented in ref. [31], we find that there are more inequivalent models in the SO(32) heterotic string for $\mathbb{Z}_N$ orbifolds with $N \leq 7$ and, conversely, the number of inequivalent models for orbifolds with $N > 7$ is larger in the case of $\mathrm{E}_8 \times \mathrm{E}_8'$. This difference becomes important if Wilson lines are present, since then one can interpret the action of the shift plus the associated Wilson line(s) locally around every fixed point as a new shift. However, this new shift must be one of the set of

[5] More details are available from the authors upon request.

JHEP04(2006)050

$\mathbb{Z}_N$	# ineq. models in SO(32)	$\mathrm{E}_8 \times \mathrm{E}_8'$
$\mathbb{Z}_3$	6	5
$\mathbb{Z}_4$	16	12
$\mathbb{Z}_6$-I	80	58
$\mathbb{Z}_6$-II	75	61
$\mathbb{Z}_7$	56	40
$\mathbb{Z}_8$-I	196	246
$\mathbb{Z}_8$-II	194	248
$\mathbb{Z}_{12}$-I	2295	3026
$\mathbb{Z}_{12}$-II	2223	3013

Table 1: Comparison between the number of inequivalent $\mathbb{Z}_N$ orbifold models in the SO(32) heterotic string and in the $\mathrm{E}_8 \times \mathrm{E}_8'$ heterotic string [31].

	# models with	# models with	
$\mathbb{Z}_N$	anomalous U_1	**16** of SO(10)	**32** of SO(12)
$\mathbb{Z}_3$	5	0	0
$\mathbb{Z}_4$	12	2	0
$\mathbb{Z}_6$-I	76	4	4
$\mathbb{Z}_6$-II	65	10	3
$\mathbb{Z}_7$	55	2	0
$\mathbb{Z}_8$-I	193	12	0
$\mathbb{Z}_8$-II	166	11	7
$\mathbb{Z}_{12}$-I	2269	80	36
$\mathbb{Z}_{12}$-II	2097	116	10

Table 2: Numbers of inequivalent $\mathbb{Z}_N$ orbifold models of the SO(32) heterotic string containing at least one spinor of SO(10) or SO(12). Spinors of bigger groups do not appear in orbifold models of the SO(32) heterotic string. We also present the number of models having an anomalous U_1 factor as part of the gauge group.

inequivalent shift vectors we have before Wilson lines are switched on[6]. In this sense, for $N \leq 7$ the SO(32) orbifolds lead to a richer variety of models.

In the second column of table 2 we present the number of models having an anomalous U_1. As explained in section 2.2, all U_1 factors are consistent with the anomaly conditions, eq. (2.10). Most of the orbifold models of the SO(32) heterotic string contain an anomalous U_1.

From the phenomenological point of view, the SO(32) heterotic string has been considered to be a less promising starting point than the $\mathrm{E}_8 \times \mathrm{E}_8'$ theory, one of the reasons being that one did not expect spinor representations to be present in the spectrum. As first shown by ref. [4], it is possible to obtain spinor representations in orbifold models of the SO(32) heterotic string from the twisted sectors. In the third and fourth columns of

[6]For more information about the concept of fixed point equivalent models, see ref. [16].

table 2, we list the number of models for each $\mathbb{Z}_N$ orbifold in which there is at least one **16** spinor of SO(10) or one **32** spinor of SO(12), respectively. As we will explain in the next section, the mass formula forbids the appearance of spinors of SO(14) or bigger groups in orbifold models of the SO(32) heterotic string.

3. Spinors in SO(32) orbifold models

In the light of recent developments, SO(10) GUTs are attractive candidates for a theory beyond the Standard Model [9 − 11]. In orbifold constructions, GUTs may be realized in an intermediate picture [5, 32 − 35], keeping their successful predictions and avoiding the problems, from which GUTs in 4 dimensions generically suffer. Therefore, we are naturally led to look for orbifold models containing the spinor of SO(10). The SO(10) gauge group can then be broken to the Standard Model gauge group by the inclusion of Wilson lines.

We investigate here the possibility of having the 16-dimensional spinor representation of SO(10). In the standard basis, the simple roots of SO(10) can be written as

$$\begin{array}{ll} \alpha_1 = (1,-1,\ \ 0,\ \ 0,\ \ 0) & \alpha_4 = (0,\ \ 0,\ \ 0,\ \ 1,-1) \\ \alpha_2 = (0,\ \ 1,-1,\ \ 0,\ \ 0) & \alpha_5 = (0,\ \ 0,\ \ 0,\ \ 1,\ \ 1) \\ \alpha_3 = (0,\ \ 0,\ \ 1,-1,\ \ 0) & \end{array}$$

In this basis, the highest weight of the **16** is given by the 5-dimensional vector $\left(\frac{1}{2}^5\right)$. This vector must be part of a 16-dimensional vector p_{sh} as follows

$$p_{sh} = p + mV = \left(\frac{1}{2}^5, a_1, a_2, \ldots, a_{11}\right), \tag{3.1}$$

where $p \in \Lambda_{\mathrm{Spin}(32)/\mathbb{Z}_2}$, V is a shift, $m \in \mathbb{N}$ is the number of the studied sector, and the numbers a_i are selected so that p_{sh} fulfills $Np_{sh} \in \Lambda_{\mathrm{Spin}(32)/\mathbb{Z}_2}$ and the mass formula for massless states

$$p_{sh}^2 = 2(1 - \tilde{N} - \delta c), \tag{3.2}$$

where δc is the shift of the zero point energy, and $\tilde{N}$ is the number operator, as explained in ref. [5]. It is important to notice that there can be more than one combination of different a_i's for which the resulting p_{sh} fulfills all the conditions.

The first consequence of the form of p_{sh} eq. (3.1) is that one cannot get the **16** of SO(10) in the untwisted sector ($m = 0$), since it only consists of the roots of SO(32), which can be expressed by the 480 vectors $(\underline{\pm 1, \pm 1, 0^{14}})$.

As a second consequence, one finds that it is not possible to get the **16** of SO(10) in the $\mathbb{Z}_3$ orbifold. The first five entries of $3p_{sh}$ are half-integer and thus, since $3p_{sh} \in \Lambda_{\mathrm{Spin}(32)/\mathbb{Z}_2}$, the remaining 11 entries must also be half-integer, i.e. $3a_i \in \mathbb{Z} + \frac{1}{2}$. Assuming the smallest value $3a_i = \frac{1}{2}$, it follows that $p_{sh}^2 \geq \frac{5}{4} + \frac{11}{36} = \frac{14}{9}$. In the case of $\tilde{N} = 0$, for any twisted sector the right-hand side of eq. (3.2) is equal to $\frac{4}{3} < \frac{14}{9}$, which forbids the appearance of p_{sh} in the spectrum of any $\mathbb{Z}_3$ orbifold model. This does not change for $\tilde{N} \neq 0$ since the value of the right-hand side of eq. (3.2) in this case is even smaller.

JHEP04(2006)050

In particular, one can easily find all shift vectors which produce SO(10) spinors in the first twisted sector. From eq. (3.1), for all $\mathbb{Z}_N$ orbifolds with $N > 3$ the shift(s) giving rise to the **16** of SO(10) in the first twisted sector ($m = 1$) can be written simply as

$$V = p_{sh} - p \xrightarrow{p=0} p_{sh}, \tag{3.3}$$

where we have chosen $p = 0$ because two shifts are equivalent if they differ by an arbitrary lattice vector. This shift is automatically modular invariant.

Finding the highest weight of the **16** of SO(10) is a necessary condition for its existence in the spectrum, but it is not sufficient to guarantee the presence of an SO(10) gauge group. One also needs to compute the gauge group induced by eq. (3.3), which can be done by simply using the patterns given in appendix B.

As an example, we consider the $\mathbb{Z}_4$ orbifold. The only possible shift consistent with $4V \in \Lambda_{\mathrm{Spin}(32)/\mathbb{Z}_2}$ and eqs. (3.2) and (3.3) with $\delta c = \frac{5}{16}$ is

$$V = p_{sh} = \left(\left(\frac{1}{2}\right)^5, \left(\frac{1}{4}\right)^2, 0^9\right). \tag{3.4}$$

This shift is equivalent to $\mathrm{V}^{(5)}$ of table 4 up to lattice vectors and Weyl reflections. One can also verify that this shift provides indeed several copies of the **16** of SO(10) in the first twisted sector.

It is phenomenologically attractive to have **16**'s of SO(10) in the first twisted sector. Since the **16**-plets of the first twisted sector are localized in all six compact dimensions, the inclusion of Wilson lines will break the gauge group and will reduce the degeneracy of the fixed points, but it will not project out parts of a **16**-plet. Therefore, models with this feature can lead to potentially realistic string models.

An indirect method to obtain the **16** of SO(10) is to switch on Wilson lines in models having spinor representations of bigger groups, like SO(12). In general, for SO($2n$) groups, the highest weight of the corresponding spinor is a solution of the eq. (3.2) of the form

$$p_{sh} = p + mV = \left(\frac{1}{2}^n, a_1, a_2, \ldots, a_{16-n}\right). \tag{3.5}$$

By inspecting all possible values that δc and $\tilde{N}$ can take in the twisted sectors for all $\mathbb{Z}_N$ orbifolds, one can see that $p_{sh}^2 = 2(1 - \tilde{N} - \delta c) \leq \frac{31}{18}$ in the mass equation (3.2). This means that the spinor representation of SO($2n$) for $n \geq 7$ given by eq. (3.5) is not allowed, because it is forbidden by the mass equation.

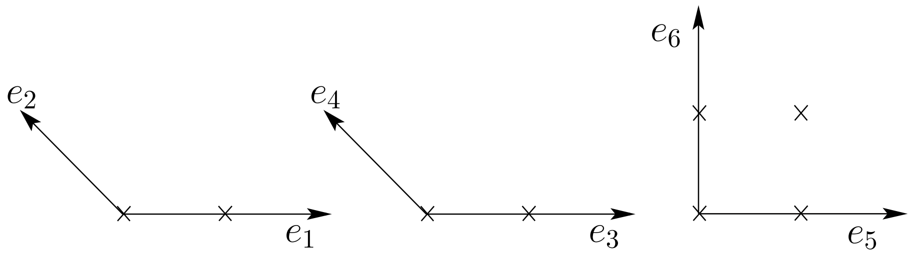
There are indeed some models with the **32** spinor representation of SO(12), as shown in table 2. One might switch on Wilson lines on them in search of realistic models.

4. Three-family orbifold models

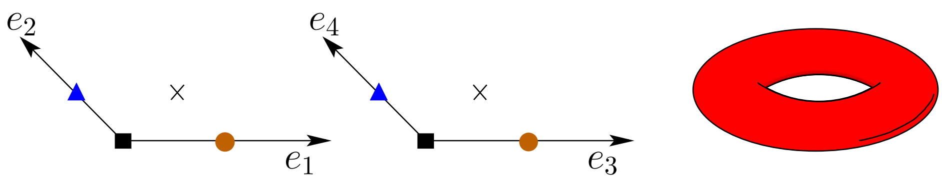
4.1 The $\mathbb{Z}_4$ orbifold

As our illustrative example, we consider the $\mathbb{Z}_4$ orbifold. One choice for the 6 dimensional lattice [27] is the $\mathrm{SO}(5)^2{\times}\mathrm{SO}(4)$ root lattice as shown in figure 3. The point group $\mathbb{Z}_4$ is generated by θ which acts as a simultaneous rotation of 90° in two of the three 2-tori and

JHEP04(2006)050



(a) θ^1 twisted sector.



(b) θ^2 twisted sector.

Figure 3: Twisted sectors of the $\mathbb{Z}_4$ orbifold.

a rotation of 180° in the third one; this corresponds to the twist vector

$$v = \frac{1}{4}\,(1,\ \ 1,\ \ -2)\,. \tag{4.1}$$

Fixed point structure. On the torus, the action of θ^1 has $2 \times 2 \times 4 = 16$ fixed points, see figure 3(a). The twisted sector corresponding to the action of θ^3 gives the anti-particles of the θ^1 sector, so we will not consider it separately.

The element θ^2 of the point group acts non-trivially only in two of the three complex planes, see figure 3(b). Thus, the strings are localized only in 4 of the 6 compact dimensions and are free to move in the last torus. For convenience, we shall refer to these fixed tori as fixed points. Of the $4 \times 4 = 16$ fixed points of the θ^2 sector, only

$$(\,\blacksquare,\blacksquare\,),\quad (\,\blacksquare,\bullet\,),\quad (\,\bullet,\blacksquare\,),\quad (\,\bullet,\bullet\,)$$

are also invariant under the action of θ. The remaining 12 points are pairwise related by θ and therefore form pairs

$$\begin{aligned}
&(\,\blacksquare,\blacktriangle\,)\leftrightarrow(\,\blacksquare,\times\,),\quad (\,\blacktriangle,\blacksquare\,)\leftrightarrow(\,\times,\blacksquare\,),\quad (\,\blacktriangle,\blacktriangle\,)\leftrightarrow(\,\times,\times\,),\\
&(\,\blacktriangle,\bullet\,)\leftrightarrow(\,\times,\bullet\,),\quad (\,\blacktriangle,\times\,)\leftrightarrow(\,\times,\blacktriangle\,),\quad (\,\bullet,\blacktriangle\,)\leftrightarrow(\,\bullet,\times\,).
\end{aligned}$$

In this way, these 12 fixed points of the θ^2 sector collapse to 6 by the action of the orbifold. This leaves an effective number of $4 + 6 = 10$ fixed points in the second twisted sector.

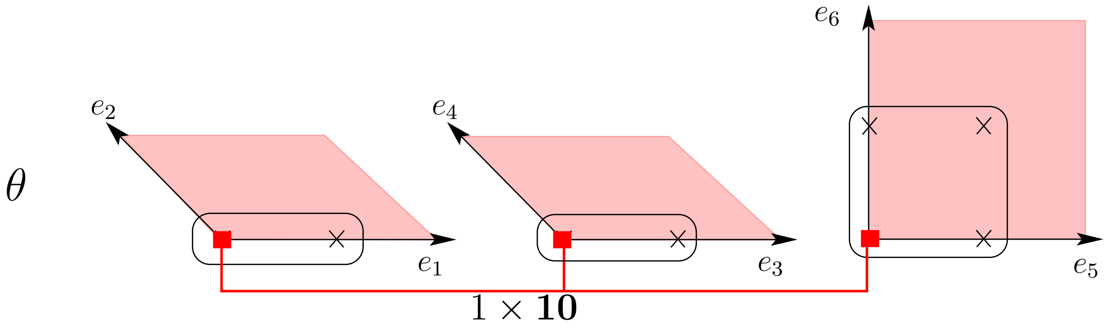


Figure 4: Localization of the three generations with SU(5) gauge group. There are two families in the bulk and one family localized in the origin. The boxes correspond to the degeneracy of the fixed points without Wilson lines. This degeneracy has been lifted by the Wilson lines in the e_2, e_4, e_5 and e_6 directions.

Wilson lines. Of the 16 $\mathbb{Z}_4$ models, none has 3 families of quarks and leptons. In order to reduce the number of families and to further break the gauge symmetry, we need Wilson lines [36]. The number and the order of Wilson lines one can add in a specific orbifold model is dictated by the geometry of the underlying compactification. In our case, we can have only 4 Wilson lines A_2, A_4, A_5, and A_6 of order 2 corresponding to the directions e_2, e_4, e_5, and e_6, respectively.

Three-generation model. As a toy model, we present a 3-generation SU(5) model. When considering the models presented in table 4, the shift vector $\mathrm{V}^{(14)}$ seems quite promising. This model has an SU(5) gauge symmetry and, most importantly, the localization of the generations gives some clues. There are two **10**'s in the bulk and sixteen **10**'s attached to the fixed points of the first twisted sector. We focus our attention on the **10** because the representation $\overline{\mathbf{5}}$ generically come with the same multiplicity due to anomaly cancellation in orbifold models. By a clever choice of the four Wilson lines, the degeneracy of the fixed points can be lifted, so that the number of families is reduced from 16 in the twisted sectors to 1, and both families of the untwisted sector survive, as depicted in figure 4:

$$
\begin{aligned}
A_2 &= \left(\frac{5}{2}^2,\ \ 0^5, -\frac{1}{2}^2 - 1^2,\ \ 3, -1^4 \right),\\
A_4 &= \left(-3^2,\ \ 0^5, -3^2, -2, -3, -\frac{5}{2},\ \ \frac{3}{2}, -\frac{5}{2}, -2,\ \ \frac{1}{2} \right),\\
A_5 &= \left(\frac{1}{2}^2,\ \ 0^5,\ \ \frac{1}{2}^2,\ \ 2,\ \ \frac{3}{2}^3,\ \ 2,\ \ \frac{1}{2},\ \ 2 \right),\\
A_6 &= \left(3,\ \ \frac{7}{2},\ \ 0^5, -1, -\frac{5}{2}, -2, -\frac{5}{2}^6 \right).
\end{aligned}
$$

The combined action of the shift and the Wilson lines leads to the gauge group $\mathrm{SU}(5) \times \mathrm{SU}(2)^5 \times \mathrm{U}(1)^7$, where the first U(1) is anomalous. The complete spectrum of this model is given in table 3. The main objective of the present publication being the clarification of some outstanding issues in the heterotic SO(32) theory, we will not explore the phenomenology of this model in detail.

U	θ^1	θ^2	sum	states
2	1		3	$(\mathbf{10};\ \mathbf{1},\ \mathbf{1},\ \mathbf{1},\ \mathbf{1},\ \mathbf{1})$
3	5		8	$(\ \overline{\mathbf{5}};\ \mathbf{1},\ \mathbf{1},\ \mathbf{1},\ \mathbf{1},\ \mathbf{1})$
1		4	5	$(\ \mathbf{5};\ \mathbf{1},\ \mathbf{1},\ \mathbf{1},\ \mathbf{1},\ \mathbf{1})$
10	37	4	51	$(\ \mathbf{1};\ \mathbf{1},\ \mathbf{1},\ \mathbf{1},\ \mathbf{1},\ \mathbf{1})$
	12	4	16	$(\ \mathbf{1};\ \mathbf{2},\ \mathbf{1},\ \mathbf{1},\ \mathbf{1},\ \mathbf{1})$
	12	4	16	$(\ \mathbf{1};\ \mathbf{1},\ \mathbf{2},\ \mathbf{1},\ \mathbf{1},\ \mathbf{1})$
	12	4	16	$(\ \mathbf{1};\ \mathbf{1},\ \mathbf{1},\ \mathbf{2},\ \mathbf{1},\ \mathbf{1})$
	12	4	16	$(\ \mathbf{1};\ \mathbf{1},\ \mathbf{1},\ \mathbf{1},\ \mathbf{2},\ \mathbf{1})$
	12	4	16	$(\ \mathbf{1};\ \mathbf{1},\ \mathbf{1},\ \mathbf{1},\ \mathbf{1},\ \mathbf{2})$

Table 3: The spectrum of a $\mathbb{Z}_4$ toy model with 3 generations of SU(5).

We would like to stress three features of this model. To the best of our knowledge, this is the first three-generation model in the context of the SO(32) orbifold published in the literature. The shift $\mathrm{V}^{(14)}$ that we used for this three-family model does not appear in ref. [4]. This model shows clearly the possibility to compute promising models through orbifolds of the SO(32) heterotic string.

4.2 Model in the $\mathbb{Z}_6$-II orbifold

In the $\mathbb{Z}_6$-II orbifold, one possible choice of the 6 dimensional lattice is $\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)$. For further details on the geometry and the fixed point structure, see ref. [33]. Even without the inclusion of Wilson lines, we find toy models with 3 generations. For instance, using

$$V^{(30)} = \left(\frac{1}{2}^2,\ -\frac{1}{6}^5,\ -\frac{1}{3}^6,\ -\frac{1}{2}^3 \right)$$

of those $\mathbb{Z}_6$-II shifts listed on our web page [8], we obtain a model with 3 generations of SO(10). Their localization is illustrated in figure 5.

The families are localized as follows: there are three **16**'s of SO(10) in the second twisted sector, whereas there are six $\overline{\mathbf{16}}$'s in the fourth twisted sector. Since the families are located in the second and fourth twisted sectors, where two of the six compactified dimensions are left invariant by the orbifold action, the families are free to move in six dimensions.

Even though this model is not realistic, it illustrates how easily one can obtain orbifold models with three families in the context of the SO(32) heterotic string. Therefore, using Wilson lines, potentially realistic models may be derived.

5. Conclusions and outlook

As we have seen, model building with the heterotic SO(32) theory might be as exciting as that with its more famous brother: the $\mathrm{E}_8 \times \mathrm{E}_8'$ string, see e.g. [37, 32–34, 38, 35, 39, 40]. It opens new roads for explicit constructions that should be explored as a vital part of the string landscape.

JHEP04(2006)050

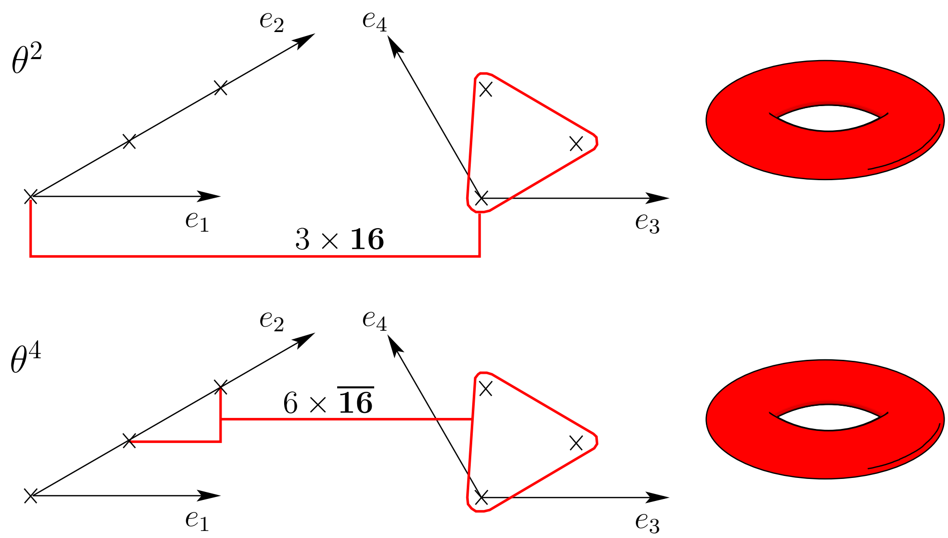


Figure 5: Localization of the 3 generations with SO(10) gauge group. There are 3 families in the second twisted sector and 6 anti-families in the fourth twisted sector, giving a net number of 3 (anti-)families free to move in two of the six compactified dimensions. The box corresponds to the degeneracy of the fixed points in the SU(3) 2-torus.

We were somewhat surprised about the frequency of the appearance of spinor representations of SO($2n$) gauge groups. These spinors might be an important tool to implement the family structure of SU(3) $\times$ SU(2) $\times$ U(1) models. In addition they are an important ingredient for a possible understanding of the SO(32) heterotic type I duality in $d = 4$ space time dimensions. We know that these spinors do not appear in the perturbative type I theory. Thus the mentioned duality will need the implementation of nonperturbative effects.

Our classification of the $\mathbb{Z}_N$ orbifolds of the SO(32) theory completes a basic building block for further model constructions. We understand this as a contribution to the study of the string landscape in the spirit of the "String Vacuum Project" [29]. A further step in this program would be the implementation of Wilson lines that leads to enormous complexity and a huge number of models (comparable to that of the $\mathrm{E}_8 \times \mathrm{E}_8'$ string). An exploration of this large region of the landscape is currently beyond our capabilities. We therefore gather our present results and make them available to the public on our web page [8], such that interested people could share our knowledge and contribute to the enterprise.

Acknowledgments

It is a pleasure to thank K.S. Choi, S. Förste and M. Ratz for valuable discussions. This work was partially supported by the European Union 6th Framework Program MRTN-CT-2004-503369 "Quest for Unification" and MRTN-CT-2004-005104 "ForcesUniverse". A.W. was supported in part by DOE grant DOE/ER/01545-866.

JHEP04(2006)050

A. Table of $\mathbb{Z}_4$ orbifold models

#	Shift	Ref. [4] classification	4D gauge group	Untwisted matter	Twisted matter T_1	Twisted matter T_2
1	$\left(0^2, \text{-}\frac{1}{2}, \text{-}\frac{3}{4}^2, 1, 0^{10}\right)$	$n_1 = 2, n_2 = 1$ Vectorial shift	$SO_{26}\times SU_2\times U_{1A}\times U_1$	$1(\mathbf{26},\mathbf{1})_{\text{-}\frac{1}{2},\text{-}\frac{1}{2}}+$ $1(\mathbf{26},\mathbf{1})_{\frac{1}{2},\frac{1}{2}}+$ $2(\mathbf{26},\mathbf{2})_{\text{-}\frac{1}{2},\frac{1}{4}}+$ $2(\mathbf{1},\mathbf{2})_{0,\text{-}\frac{3}{4}}+2(\mathbf{1},\mathbf{2})_{1,\frac{1}{4}}+$ $1(\mathbf{1},\mathbf{1})_{1,\text{-}\frac{1}{2}}+1(\mathbf{1},\mathbf{1})_{\text{-}1,\frac{1}{2}}$	$16(\mathbf{26},\mathbf{1})_{\text{-}\frac{1}{2},\text{-}\frac{1}{8}}+$ $32(\mathbf{1},\mathbf{2})_{0,\text{-}\frac{3}{8}}+$ $16(\mathbf{1},\mathbf{1})_{1,\text{-}\frac{1}{8}}+$ $80(\mathbf{1},\mathbf{1})_{0,\frac{3}{8}}$	$10(\mathbf{26},\mathbf{1})_{\text{-}\frac{1}{2},\frac{1}{4}}+$ $6(\mathbf{26},\mathbf{1})_{\frac{1}{2},\text{-}\frac{1}{4}}+$ $32(\mathbf{1},\mathbf{2})_{0,0}+$ $10(\mathbf{1},\mathbf{1})_{0,\text{-}\frac{3}{4}}+$ $10(\mathbf{1},\mathbf{1})_{1,\frac{1}{4}}+$ $6(\mathbf{1},\mathbf{1})_{\text{-}1,\text{-}\frac{1}{4}}+6(\mathbf{1},\mathbf{1})_{0,\frac{3}{4}}$
2	$\left(0^2, \text{-}\frac{1}{2}^2, \frac{1}{2}, \frac{1}{4}, \text{-}\frac{3}{4}, 1, 0^8\right)$	$n_1 = 2, n_2 = 3$ Vectorial shift	$SO_{22}\times SU_4\times SU_2\times U_1$	$1(\mathbf{22},\mathbf{6},\mathbf{1})_0+$ $2(\mathbf{22},\mathbf{1},\mathbf{2})_{\frac{1}{4}}+$ $2(\mathbf{1},\mathbf{6},\mathbf{2})_{\text{-}\frac{1}{4}}+$ $1(\mathbf{1},\mathbf{1},\mathbf{1})_{\text{-}\frac{1}{2}}+1(\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{1}{2}}$	$16(\mathbf{1},\overline{\mathbf{4}},\mathbf{2})_{\text{-}\frac{1}{8}}+$ $32(\mathbf{1},\mathbf{4},\mathbf{1})_{\frac{1}{8}}$	$10(\mathbf{22},\mathbf{1},\mathbf{1})_{\text{-}\frac{1}{4}}+$ $6(\mathbf{22},\mathbf{1},\mathbf{1})_{\frac{1}{4}}+$ $10(\mathbf{1},\mathbf{6},\mathbf{1})_{\frac{1}{4}}+$ $6(\mathbf{1},\mathbf{6},\mathbf{1})_{\text{-}\frac{1}{4}}+32(\mathbf{1},\mathbf{1},\mathbf{2})_0$
3	$\left(0^2, \text{-}\frac{3}{4}^2, \frac{1}{4}^3, \frac{9}{4}, \text{-}2, 0^7\right)$	$n_1 = 6, n_2 = 0$ Vectorial shift	$SO_{20}\times SU_6\times U_{1A}$	$2(\mathbf{20},\overline{\mathbf{6}})_{\text{-}\frac{1}{2}}+$ $1(\mathbf{1},\mathbf{15})_1+1(\mathbf{1},\overline{\mathbf{15}})_{\text{-}1}$	$16(\mathbf{1},\mathbf{15})_{\frac{1}{4}}+$ $80(\mathbf{1},\mathbf{1})_{\text{-}\frac{3}{4}}$	$10(\mathbf{1},\mathbf{15})_{\text{-}\frac{1}{2}}+6(\mathbf{1},\overline{\mathbf{15}})_{\frac{1}{2}}+$ $10(\mathbf{1},\mathbf{1})_{\frac{3}{2}}+6(\mathbf{1},\mathbf{1})_{\text{-}\frac{3}{2}}$
4	$\left(0^2, \text{-}\frac{1}{4}, \text{-}\frac{3}{4}, \frac{1}{4}^3, \text{-}\frac{3}{4}, 1, 0^7\right)$	not classified Vectorial shift	$SO_{20}\times SU_6\times U_{1A}$	$2(\mathbf{20},\mathbf{6})_{\text{-}\frac{1}{2}}+$ $1(\mathbf{1},\mathbf{15})_{\text{-}1}+1(\mathbf{1},\overline{\mathbf{15}})_1$	$16(\mathbf{20},\mathbf{1})_{\text{-}\frac{3}{4}}+$ $32(\mathbf{1},\overline{\mathbf{6}})_{\text{-}\frac{1}{4}}$	$10(\mathbf{1},\overline{\mathbf{15}})_{\text{-}\frac{1}{2}}+6(\mathbf{1},\mathbf{15})_{\frac{1}{2}}+$ $10(\mathbf{1},\mathbf{1})_{\frac{3}{2}}+6(\mathbf{1},\mathbf{1})_{\text{-}\frac{3}{2}}$
5	$\left(0^2, \text{-}\frac{1}{2}^2, \frac{1}{2}^3, \frac{1}{4}, \frac{9}{4}, \text{-}2, 0^6\right)$	$n_1 = 2, n_2 = 5$ Vectorial shift	$SO_{18}\times SO_{10}\times SU_2\times U_{1A}$	$1(\mathbf{18},\mathbf{10},\mathbf{1})_0+$ $2(\mathbf{18},\mathbf{1},\mathbf{2})_{\text{-}\frac{1}{2}}+$ $2(\mathbf{1},\mathbf{10},\mathbf{2})_{\frac{1}{2}}+$ $1(\mathbf{1},\mathbf{1},\mathbf{1})_1+1(\mathbf{1},\mathbf{1},\mathbf{1})_{\text{-}1}$	$16(\mathbf{1},\mathbf{16},\mathbf{1})_{\text{-}\frac{1}{4}}$	$10(\mathbf{18},\mathbf{1},\mathbf{1})_{\text{-}\frac{1}{2}}+$ $6(\mathbf{18},\mathbf{1},\mathbf{1})_{\frac{1}{2}}+$ $10(\mathbf{1},\mathbf{10},\mathbf{1})_{\frac{1}{2}}+$ $6(\mathbf{1},\mathbf{10},\mathbf{1})_{\text{-}\frac{1}{2}}+$ $32(\mathbf{1},\mathbf{1},\mathbf{2})_0$

Table 4: *Continued...*

#	Shift	Ref. [4] classification	4D gauge group	Untwisted matter	Twisted matter	
					T_1	T_2
6	$\left(0^2, \text{-}\frac{1}{2}^2, \frac{1}{4}^5, \text{-}\frac{3}{4}, 1, 0^5\right)$	$n_1=6, n_2=2$ Vectorial shift	$\mathrm{SO}_{16}\times\ \mathrm{SU}_2^2\times\ \mathrm{SU}_6\times\ \mathrm{U}_{1A}$	$1(\mathbf{16},\mathbf{2},\mathbf{2},\mathbf{1})_0\ +$ $2(\mathbf{16},\mathbf{1},\mathbf{1},\overline{\mathbf{6}})_{\text{-}\frac{1}{2}}\ +$ $1(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{15})_1\ +$ $1(\mathbf{1},\mathbf{1},\mathbf{1},\overline{\mathbf{15}})_{\text{-}1}\ +$ $2(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{6})_{\frac{1}{2}}$	$16(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{6})_{\text{-}\frac{1}{4}}\ +$ $32(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})_{\text{-}\frac{3}{4}}$	$10(\mathbf{1},\mathbf{1},\mathbf{1},\overline{\mathbf{15}})_{\frac{1}{2}}\ +$ $6(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{15})_{\text{-}\frac{1}{2}}\ +$ $10(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{\text{-}\frac{3}{2}}\ +$ $6(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{3}{2}}$
7	$\left(0^2, \text{-}\frac{1}{2}^4, \frac{1}{2}^3, \frac{1}{4}, \frac{5}{4}, \text{-}1, 0^4\right)$	$n_1=2, n_2=7$ Vectorial shift	$\mathrm{SO}_{14}\times\ \mathrm{SO}_{14}\times\ \mathrm{SU}_2\times\ \mathrm{U}_1$	$1(\mathbf{14},\mathbf{14},\mathbf{1})_0\ +$ $2(\mathbf{14},\mathbf{1},\mathbf{2})_{\frac{1}{4}}\ +$ $2(\mathbf{1},\mathbf{14},\mathbf{2})_{\text{-}\frac{1}{4}}\ +$ $1(\mathbf{1},\mathbf{1},\mathbf{1})_{\text{-}\frac{1}{2}} + 1(\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{1}{2}}$		$10(\mathbf{14},\mathbf{1},\mathbf{1})_{\text{-}\frac{1}{4}}\ +$ $6(\mathbf{14},\mathbf{1},\mathbf{1})_{\frac{1}{4}}\ +$ $10(\mathbf{1},\mathbf{14},\mathbf{1})_{\frac{1}{4}}\ +$ $6(\mathbf{1},\mathbf{14},\mathbf{1})_{\text{-}\frac{1}{4}}\ +$ $32(\mathbf{1},\mathbf{1},\mathbf{2})_0$
8	$\left(0^2, \text{-}\frac{1}{2}^4, \frac{1}{4}^5, \frac{9}{4}, \text{-}2, 0^3\right)$	$n_1=6, n_2=4$ Vectorial shift	$\mathrm{SO}_{12}\times\ \mathrm{SO}_{8}\times\ \mathrm{SU}_6\times\ \mathrm{U}_{1A}$	$1(\mathbf{12},\mathbf{8},\mathbf{1})_0\ +$ $2(\mathbf{12},\mathbf{1},\overline{\mathbf{6}})_{\text{-}\frac{1}{2}}\ +$ $2(\mathbf{1},\mathbf{8},\mathbf{6})_{\frac{1}{2}}\ +$ $1(\mathbf{1},\mathbf{1},\mathbf{15})_1\ +$ $1(\mathbf{1},\mathbf{1},\overline{\mathbf{15}})_{\text{-}1}$	$16(\mathbf{1},\mathbf{8},\mathbf{1})_{\text{-}\frac{3}{4}}$	$10(\mathbf{1},\mathbf{1},\mathbf{15})_{\text{-}\frac{1}{2}}\ +$ $6(\mathbf{1},\mathbf{1},\overline{\mathbf{15}})_{\frac{1}{2}}\ +$ $10(\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{3}{2}}\ +$ $6(\mathbf{1},\mathbf{1},\mathbf{1})_{\text{-}\frac{3}{2}}$
9	$\left(0^2, \text{-}\frac{1}{2}, \text{-}\frac{3}{4}, \frac{1}{4}^8, \frac{9}{4}, \text{-}2, 0^2\right)$	$n_1=10, n_2=1$ Vectorial shift	$\mathrm{SO}_{10}\times\ \mathrm{SU}_{10}\times\ \mathrm{U}_{1A}\times\mathrm{U}_1$	$2(\mathbf{10},\overline{\mathbf{10}})_{\text{-}\frac{1}{2},\frac{1}{4}}\ +$ $1(\mathbf{10},\mathbf{1})_{\text{-}1,\text{-}\frac{5}{4}}\ +$ $1(\mathbf{10},\mathbf{1})_{1,\frac{5}{4}}\ +$ $2(\mathbf{1},\mathbf{10})_{\text{-}\frac{1}{2},\text{-}\frac{3}{2}}\ +$ $2(\mathbf{1},\mathbf{10})_{\frac{3}{2},1}\ +$ $1(\mathbf{1},\mathbf{45})_{1,\text{-}\frac{1}{2}}\ +$ $1(\mathbf{1},\overline{\mathbf{45}})_{\text{-}1,\frac{1}{2}}$	$16(\mathbf{1},\mathbf{10})_{\text{-}\frac{5}{4},\text{-}\frac{1}{4}}\ +$ $32(\mathbf{1},\mathbf{1})_{\text{-}\frac{3}{4},\frac{5}{4}}$	$10(\mathbf{16},\mathbf{1})_{\text{-}\frac{1}{2},\text{-}\frac{5}{8}}\ +$ $6(\overline{\mathbf{16}},\mathbf{1})_{\frac{1}{2},\frac{5}{8}}$
10	$\left(0^2, \text{-}\frac{1}{2}^2, \frac{1}{2}, \frac{1}{4}^9, \frac{9}{4}, 2\right)$	$n_1=10, n_2=3$ Vectorial shift	$\mathrm{SU}_4\times\ \mathrm{SU}_4\times\ \mathrm{SU}_{10}\times\ \mathrm{U}_1$	$1(\mathbf{6},\mathbf{6},\mathbf{1})_0\ +$ $2(\mathbf{6},\mathbf{1},\overline{\mathbf{10}})_{\frac{1}{4}}\ +$ $2(\mathbf{1},\mathbf{6},\mathbf{10})_{\text{-}\frac{1}{4}}\ +$ $1(\mathbf{1},\mathbf{1},\mathbf{45})_{\text{-}\frac{1}{2}}\ +$ $1(\mathbf{1},\mathbf{1},\overline{\mathbf{45}})_{\frac{1}{2}}$	$16(\overline{\mathbf{4}},\mathbf{1},\mathbf{1})_{\text{-}\frac{5}{8}}\ +$ $16(\mathbf{1},\mathbf{4},\mathbf{1})_{\frac{5}{8}}$	$10(\mathbf{4},\overline{\mathbf{4}},\mathbf{1})_0 + 6(\overline{\mathbf{4}},\mathbf{4},\mathbf{1})_0$

Table 4: *Continued...*

#	Shift	Ref. [4] classification	4D gauge group	Untwisted matter	Twisted matter T_1	Twisted matter T_2
11	$\left(\frac{1}{2}^2,-\frac{1}{4}^{12},\frac{3}{4}^2\right)$	$n_1=14, n_2=0$ Vectorial shift	$\mathrm{SU}_2\times\mathrm{SU}_2\times\mathrm{SU}_{14}\times\mathrm{U}_1$	$2(\mathbf{2},\mathbf{2},\overline{\mathbf{14}})_{\frac{1}{4}}+$ $1(\mathbf{1},\mathbf{1},\mathbf{91})_{-\frac{1}{2}}+$ $1(\mathbf{1},\mathbf{1},\overline{\mathbf{91}})_{\frac{1}{2}}$	$16(\mathbf{2},\mathbf{1},\mathbf{1})_{-\frac{7}{8}}+$ $32(\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{7}{8}}$	$10(\mathbf{1},\mathbf{2},\mathbf{14})_{-\frac{1}{4}}+$ $32(\mathbf{2},\mathbf{1},\mathbf{1})_0+$ $6(\mathbf{1},\mathbf{2},\overline{\mathbf{14}})_{\frac{1}{4}}$
12	$\left(\frac{1}{2}^2,\frac{1}{4},-\frac{1}{4}^{11},\frac{3}{4}^2\right)$	not classified Vectorial shift	$\mathrm{SU}_2\times\mathrm{SU}_2\times\mathrm{SU}_{14}\times\mathrm{U}_{1A}$	$2(\mathbf{2},\mathbf{2},\mathbf{14})_{-\frac{1}{2}}+$ $1(\mathbf{1},\mathbf{1},\mathbf{91})_{-1}+$ $1(\mathbf{1},\mathbf{1},\overline{\mathbf{91}})_{1}$	$16(\mathbf{2},\mathbf{1},\mathbf{1})_{\frac{7}{4}}+$ $16(\mathbf{1},\mathbf{1},\overline{\mathbf{14}})_{-\frac{5}{4}}$	$10(\mathbf{2},\mathbf{1},\mathbf{14})_{-\frac{1}{2}}+$ $32(\mathbf{1},\mathbf{2},\mathbf{1})_0+$ $6(\mathbf{2},\mathbf{1},\overline{\mathbf{14}})_{\frac{1}{2}}$
13	$\left(-\frac{1}{8},-\frac{7}{8},-\frac{5}{8},\frac{1}{8}^{11},\frac{17}{8}^2\right)$	Spinorial shift	$\mathrm{SU}_{15}\times\mathrm{U}_{1A}\times\mathrm{U}_1$	$2\times\mathbf{105}_{-5,\frac{3}{2}}+$ $2\times\overline{\mathbf{15}}_{7,\frac{11}{2}}+\mathbf{15}_{2,7}+$ $\overline{\mathbf{15}}_{-2,-7}$	$16\times\overline{\mathbf{15}}_{-5,-\frac{13}{4}}$ $+80\times\mathbf{1}_{-3,\frac{15}{4}}$	$10\times\overline{\mathbf{15}}_{-8,\frac{1}{2}}+$ $10\times\mathbf{1}_{6,-\frac{15}{2}}+$ $6\times\mathbf{15}_{8,-\frac{1}{2}}+6\times\mathbf{1}_{-6,\frac{15}{2}}$
14	$\left(-\frac{9}{8},\frac{1}{8},-\frac{13}{8},-\frac{5}{8}^4,-\frac{7}{8}^9\right)$	Spinorial shift	$\mathrm{SU}_{11}\times\mathrm{SU}_5\times\mathrm{U}_{1A}\times\mathrm{U}_1$	$2(\mathbf{55},\mathbf{1})_{-3,-\frac{5}{2}}+$ $2(\overline{\mathbf{11}},\overline{\mathbf{5}})_{1,\frac{19}{2}}+$ $2(\mathbf{1},\mathbf{10})_{1,-\frac{33}{2}}+$ $1(\mathbf{11},\overline{\mathbf{5}})_{-2,7}+1(\overline{\mathbf{11}},\mathbf{5})_{2,-7}$	$16(\mathbf{1},\mathbf{10})_{-2,-\frac{11}{4}}+$ $32(\mathbf{1},\mathbf{1})_{-3,\frac{55}{4}}$	$10(\overline{\mathbf{11}},\mathbf{1})_{-2,-\frac{25}{2}}+$ $10(\mathbf{1},\mathbf{5})_{4,\frac{11}{2}}+$ $6(\mathbf{11},\mathbf{1})_{2,\frac{25}{2}}+$ $6(\mathbf{1},\overline{\mathbf{5}})_{-4,-\frac{11}{2}}$
15	$\left(-\frac{1}{8},\frac{1}{8},-\frac{5}{8}^4,\frac{3}{8}^5,\frac{1}{8}^5\right)$	Spinorial shift	$\mathrm{SU}_7\times\mathrm{SU}_9\times\mathrm{U}_{1A}\times\mathrm{U}_1$	$2(\mathbf{21},\mathbf{1})_{3,-\frac{3}{2}}+$ $2(\overline{\mathbf{7}},\overline{\mathbf{9}})_{-1,\frac{5}{2}}+$ $2(\mathbf{1},\mathbf{36})_{-1,-\frac{7}{2}}+$ $1(\mathbf{7},\overline{\mathbf{9}})_{2,1}+1(\overline{\mathbf{7}},\mathbf{9})_{-2,-1}$	$16(\mathbf{7},\mathbf{1})_{-3,-\frac{3}{4}}+$ $16(\mathbf{1},\mathbf{1})_{3,\frac{21}{4}}$	$10(\overline{\mathbf{7}},\mathbf{1})_{0,-\frac{9}{2}}+$ $10(\mathbf{1},\mathbf{9})_{-2,\frac{7}{2}}+$ $6(\mathbf{7},\mathbf{1})_{0,\frac{9}{2}}+6(\mathbf{1},\overline{\mathbf{9}})_{2,-\frac{7}{2}}$
16	$\left(\frac{3}{8},\frac{5}{8},-\frac{1}{8}^{12},\frac{15}{8},-\frac{3}{8}\right)$	Spinorial shift	$\mathrm{SU}_3\times\mathrm{SU}_{13}\times\mathrm{U}_{1A}\times\mathrm{U}_1$	$2(\overline{\mathbf{3}},\mathbf{1})_{3,-\frac{13}{2}}+$ $2(\overline{\mathbf{3}},\overline{\mathbf{13}})_{-1,\frac{11}{2}}+$ $2(\mathbf{1},\mathbf{78})_{-1,-\frac{9}{2}}+$ $1(\mathbf{3},\overline{\mathbf{13}})_{2,-1}+1(\overline{\mathbf{3}},\mathbf{13})_{-2,1}$	$16(\mathbf{1},\mathbf{1})_{2,-\frac{39}{4}}+$ $16(\mathbf{1},\overline{\mathbf{13}})_{-2,\frac{9}{4}}+$ $32(\mathbf{3},\mathbf{1})_{-1,-\frac{13}{4}}$	$10(\overline{\mathbf{3}},\mathbf{1})_{-2,-\frac{13}{2}}+$ $10(\mathbf{1},\mathbf{13})_{0,\frac{15}{2}}+$ $6(\mathbf{3},\mathbf{1})_{2,\frac{13}{2}}+$ $6(\mathbf{1},\overline{\mathbf{13}})_{0,-\frac{15}{2}}$

Table 4: All admissible models for the $\mathbb{Z}_4$ orbifold of the SO(32) heterotic string without Wilson lines. For each model, we list the shift vector, its classification according to ref. [4] and the matter content displayed in sectors.

B. The general form of a shift in $\mathbb{Z}_N$ orbifolds of the SO(32) heterotic string

To obtain the general form of a shift, we use the fact that two shifts are equivalent if they are related by lattice vectors or by Weyl reflections, i.e. by any permutation of the entries and pairwise sign flips.

In $\mathbb{Z}_N$ orbifolds with **even N**, one can prove that the most general form of a **vectorial shift** is given by

$$V = \frac{1}{N}\left((\pm k)^{\alpha}, -(N-k)^{\beta}, 0^{n_0}, 1^{n_1}, \dots, (N-k)^{n_{(N-k)}-\alpha-\beta}, \dots, \left(\frac{N}{2}\right)^{n_{\left(\frac{N}{2}\right)}}\right), \quad \text{(B.1)}$$

where $\alpha, \beta, n_i \in \mathbb{N}$, $\alpha + \beta \in \{0, 1\}$, $k \in \{\frac{N}{2}+1, \frac{N}{2}+2, \dots, N\}$ and $\sum n_i = 16$. It leads to a symmetry breaking in four dimensions of the general form

$$\mathrm{SO}(32) \longrightarrow \mathrm{SO}(2n_0) \times \mathrm{U}(n_1) \times \dots \times \mathrm{U}\left(n_{\left(\frac{N}{2}-1\right)}\right) \times \mathrm{SO}\left(2n_{\left(\frac{N}{2}\right)}\right). \quad \text{(B.2)}$$

On the other hand, for **even N** the **spinorial shifts** can be written in the standard form

$$V = \frac{1}{2N}\left((\pm k)^{\alpha}, -(2N-k)^{\beta}, 1^{n_1}, 3^{n_3}, \dots, (2N-k)^{n_{(2N-k)}-\alpha-\beta}, \dots, (N-1)^{n_{(N-1)}}\right), \quad \text{(B.3)}$$

with $k \in \{N+1, N+3, \dots, 2N-1\}$, which give rise to the gauge group

$$\mathrm{SO}(32) \longrightarrow \mathrm{U}(n_1) \times \mathrm{U}(n_3) \times \dots \times \mathrm{U}\left(n_{(N-3)}\right) \times \mathrm{U}\left(n_{(N-1)}\right). \quad \text{(B.4)}$$

For **N odd**, the general form of both a vectorial shift and a spinorial one changes slightly. As explained in ref. [4], in this case it is enough to determine either the vectorial or the spinorial shifts, since one spinorial shift can always be transformed into a vectorial one by the action of Weyl reflections and lattice vectors. Therefore any shift can be written in general as

$$V = \frac{1}{N}\left((\pm k)^{\alpha}, -(N-k)^{\beta}, 0^{n_0}, 1^{n_1}, \dots, (N-k)^{n_{(N-k)}-\alpha-\beta}, \dots, \left(\frac{N-1}{2}\right)^{n_{\left(\frac{N-1}{2}\right)}}\right), \quad \text{(B.5)}$$

where $k \in \{\frac{N+1}{2}, \frac{N+3}{2}, \dots, N\}$. The resulting four dimensional gauge group is

$$\mathrm{SO}(32) \longrightarrow \mathrm{SO}(2n_0) \times \mathrm{U}(n_1) \times \dots \times \mathrm{U}\left(n_{\left(\frac{N-3}{2}\right)}\right) \times \mathrm{U}\left(n_{\left(\frac{N-1}{2}\right)}\right). \quad \text{(B.6)}$$

References

[1] J. Polchinski and E. Witten, *Evidence for heterotic–type-I string duality*, *Nucl. Phys.* **B 460** (1996) 525 [hep-th/9510169].

[2] L.J. Dixon, J.A. Harvey, C. Vafa and E. Witten, *Strings on orbifolds*, *Nucl. Phys.* **B 261** (1985) 678.

[3] J. Giedt, *Z_3 orbifolds of the SO(32) heterotic string. 1 Wilson line embeddings*, *Nucl. Phys.* **B 671** (2003) 133 [hep-th/0301232].

[4] K.-S. Choi, S. Groot Nibbelink and M. Trapletti, *Heterotic SO(32) model building in four dimensions*, *JHEP* **12** (2004) 063 [hep-th/0410232].

[5] S. Förste, H.P. Nilles, P.K.S. Vaudrevange and A. Wingerter, *Heterotic brane world*, *Phys. Rev.* **D 70** (2004) 106008 [hep-th/0406208].

[6] S. Förste, H.P. Nilles and A. Wingerter, *Geometry of rank reduction*, *Phys. Rev.* **D 72** (2005) 026001 [hep-th/0504117].

[7] S. Förste, H.P. Nilles and A. Wingerter, *The Higgs mechanism in heterotic orbifolds*, *Phys. Rev.* **D 73** (2006) 066011 [hep-th/0512270].

[8] H.P. Nilles, S. Ramos-Sánchez, P.K.S. Vaudrevange and A. Wingerter, http://www.th.physik.uni-bonn.de/nilles/orbifolds.

[9] H. Georgi, *The state of the art — gauge theories (talk)*, *AIP Conf. Proc.* **23** (1975) 575

[10] H. Fritzsch and P. Minkowski, *Unified interactions of leptons and hadrons*, *Ann. Phys. (NY)* **93** (1975) 193.

[11] H.P. Nilles, *Five golden rules for superstring phenomenology*, hep-th/0410160.

[12] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y.S. Stanev, *Chiral asymmetry in four-dimensional open- string vacua*, *Phys. Lett.* **B 385** (1996) 96 [hep-th/9606169].

[13] Z. Kakushadze, *Aspects of $N = 1$ type-I-heterotic duality in four dimensions*, *Nucl. Phys.* **B 512** (1998) 221 [hep-th/9704059].

[14] Z. Lalak, S. Lavignac and H.P. Nilles, *String dualities in the presence of anomalous U(1) symmetries*, *Nucl. Phys.* **B 559** (1999) 48 [hep-th/9903160].

[15] R. Blumenhagen, G. Honecker and T. Weigand, *Supersymmetric (non-)abelian bundles in the type-I and SO(32) heterotic string*, *JHEP* **08** (2005) 009 [hep-th/0507041].

[16] F. Gmeiner, S. Groot Nibbelink, H.P. Nilles, M. Olechowski and M.G.A. Walter, *Localized anomalies in heterotic orbifolds*, *Nucl. Phys.* **B 648** (2003) 35 [hep-th/0208146].

[17] L.E. Ibáñez, J. Mas, H.P. Nilles and F. Quevedo, *Heterotic strings in symmetric and asymmetric orbifold backgrounds*, *Nucl. Phys.* **B 301** (1988) 157.

[18] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory. vol. 1. Introduction*, app. 5A.

[19] R. Slansky, *Group theory for unified model building*, *Phys. Rept.* **79** (1981) 1.

[20] L.J. Dixon, J.A. Harvey, C. Vafa and E. Witten, *Strings on orbifolds, 2*, *Nucl. Phys.* **B 274** (1986) 285.

[21] V.G. Kac, *Automorphisms of finite order of semisimple Lie algebras*, *Func. Anal. Appl.* **3** (1969) 252.

[22] M.B. Green and J.H. Schwarz, *Anomaly cancellation in supersymmetric $D = 10$ gauge theory and superstring theory*, *Phys. Lett.* **B 149** (1984) 117.

[23] E. Witten, *Some properties of O(32) superstrings*, *Phys. Lett.* **B 149** (1984) 351.

[24] M. Dine, N. Seiberg and E. Witten, *Fayet-Iliopoulos terms in string theory*, *Nucl. Phys.* **B 289** (1987) 589.

[25] A. Sagnotti, *A note on the Green-Schwarz mechanism in open string theories*, *Phys. Lett.* **B 294** (1992) 196 [hep-th/9210127].

[26] M. Berkooz et al., *Anomalies, dualities and topology of $D = 6$ $N = 1$ superstring vacua*, *Nucl. Phys.* **B 475** (1996) 115 [hep-th/9605184].

[27] T. Kobayashi and N. Ohtsubo, *Geometrical aspects of Z(N) orbifold phenomenology*, *Int. J. Mod. Phys.* **A 9** (1994) 87.

[28] J.A. Casas, F. Gomez and C. Muñoz, *Complete structure of Z(N) Yukawa couplings*, *Int. J. Mod. Phys.* **A 8** (1993) 455 [hep-th/9110060].

[29] The European String Vacuum Project website is located at http://www.ippp.dur.ac.uk/∼dgrell/svp/.

[30] K.R. Dienes, *Statistics on the heterotic landscape: gauge groups and cosmological constants of four-dimensional heterotic strings*, hep-th/0602286.

[31] Y. Katsuki et al., *Z(N) orbifold models*, *Nucl. Phys.* **B 341** (1990) 611;
Y. Katsuki, Y. Kawamura, T. Kobayashi, N. Ohtsubo, Y. Ono and K. Tanioka, *Tables of Z(N) orbifold models*, DPKU-8904.

[32] T. Kobayashi, S. Raby and R.J. Zhang, *Constructing 5D orbifold grand unified theories from heterotic strings*, *Phys. Lett.* **B 593** (2004) 262 [hep-ph/0403065].

[33] T. Kobayashi, S. Raby and R.-J. Zhang, *Searching for realistic 4D string models with a pati-Salam symmetry: orbifold grand unified theories from heterotic string compactification on a Z(6) orbifold*, *Nucl. Phys.* **B 704** (2005) 3 [hep-ph/0409098].

[34] W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, *Dual models of gauge unification in various dimensions*, *Nucl. Phys.* **B 712** (2005) 139 [hep-ph/0412318].

[35] W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, *The supersymmetric standard model from the heterotic string*, hep-ph/0511035.

[36] L.E. Ibáñez, H.P. Nilles and F. Quevedo, *Orbifolds and Wilson lines*, *Phys. Lett.* **B 187** (1987) 25.

[37] A.E. Faraggi, D.V. Nanopoulos and K.J. Yuan, *A standard like model in the 4D free fermionic string formulation*, *Nucl. Phys.* **B 335** (1990) 347.

[38] V. Braun, Y.-H. He, B.A. Ovrut and T. Pantev, *A heterotic standard model*, *Phys. Lett.* **B 618** (2005) 252 [hep-th/0501070].

[39] V. Bouchard and R. Donagi, *An* SU(5) *heterotic standard model*, *Phys. Lett.* **B 633** (2006) 783 [hep-th/0512149].

[40] R. Blumenhagen, S. Moster and T. Weigand, *Heterotic GUT and standard model vacua from simply connected Calabi-Yau manifolds*, hep-th/0603015.